Quarkonium theory

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Department of Mathematics and Physics
University of Stavanger

References:

S.Kim, P. Petreczky, A.R., JHEP 1811 (2018) 088

P. Petreczky, A.R., J. Weber, NPA982 (2019) 735

D. Lafferty, A.R. arXiv:1906.00035

Y. Akamatsu et. al. JHEP 1807 (2018) 029 & (in preparation)

A challenge to theory

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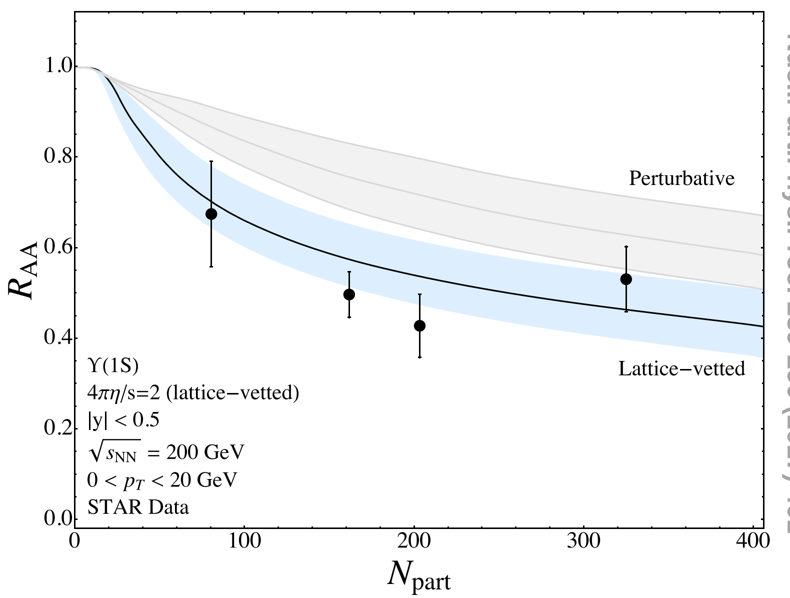
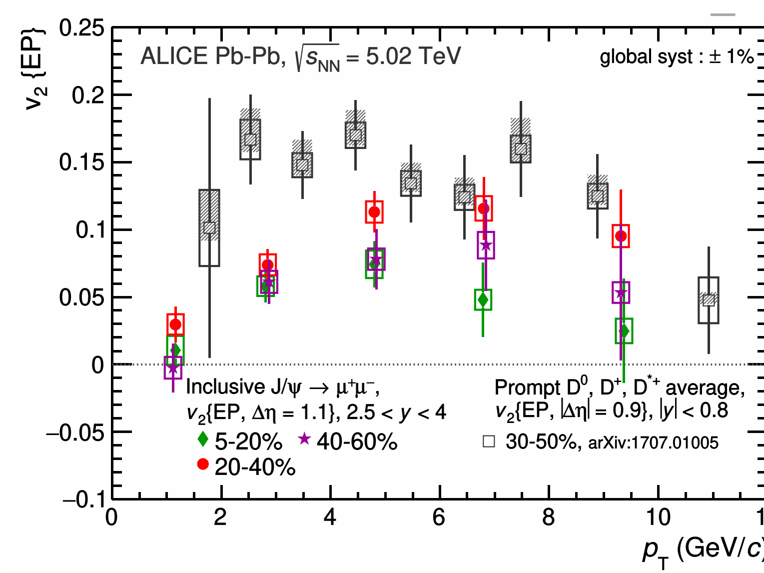
- A wealth of high precision data on both flavors from RHIC and LHC



Kroupa et. al., PRDD97 (2018) 016017
[STAR Collaboration] Z. Ye Nucl.Part.Phys.Proc. 289-290 (2017) 401



[ALICE collaboration] PRL119 (2017) 242301

Bottomonium: a non-equilibrium probe of the full QGP evolution

Charmonium: a partially equilibrated probe, sensitive to the late stages

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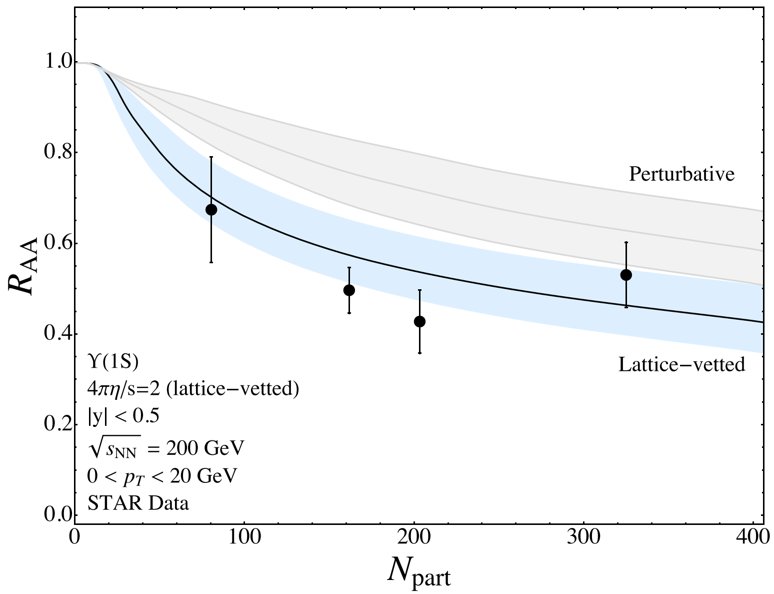

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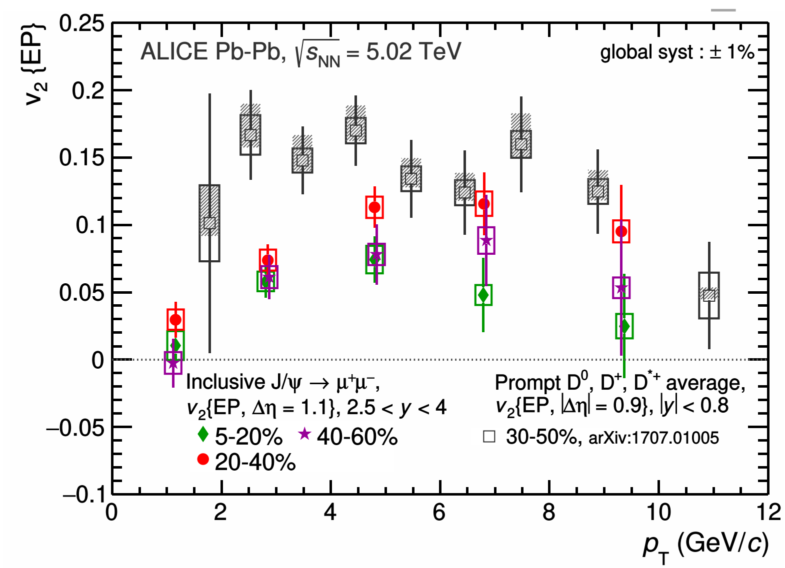

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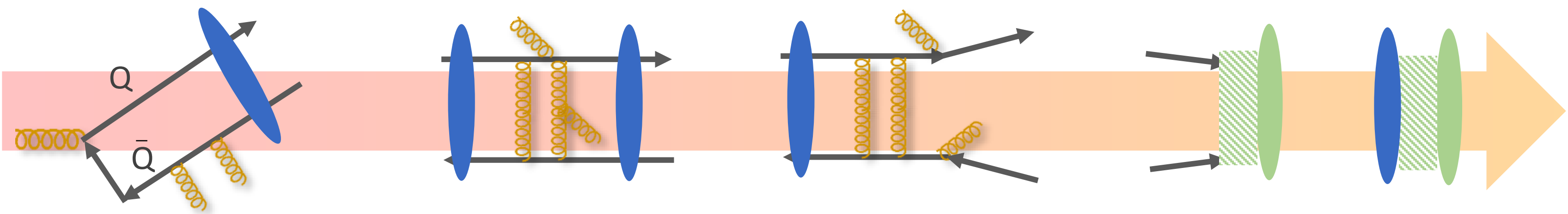
Bottomonium: a non-equilibrium probe of the full QGP evolution

Charmonium: a partially equilibrated probe, sensitive to the late stages

- Goal: provide first principles interpretation to intricate phenomenology

Selected theory goals



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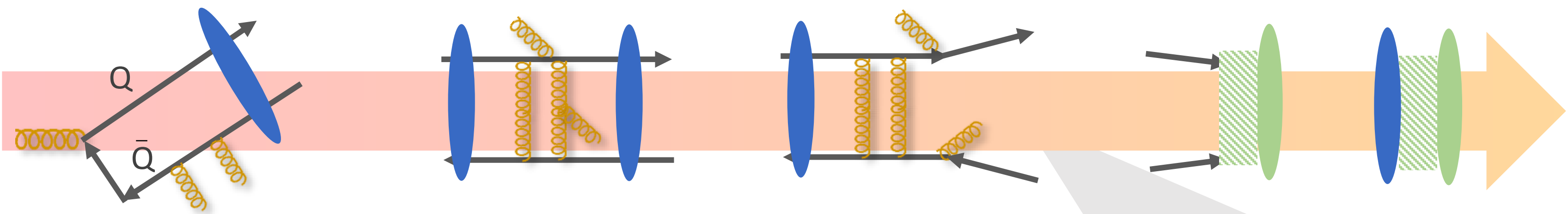

Properties of equilibrium QQ

Robust in-medium masses from lattice NRQCD

with S.Kim, P. Petreczky: JHEP 1811 (2018) 088

Selected theory goals

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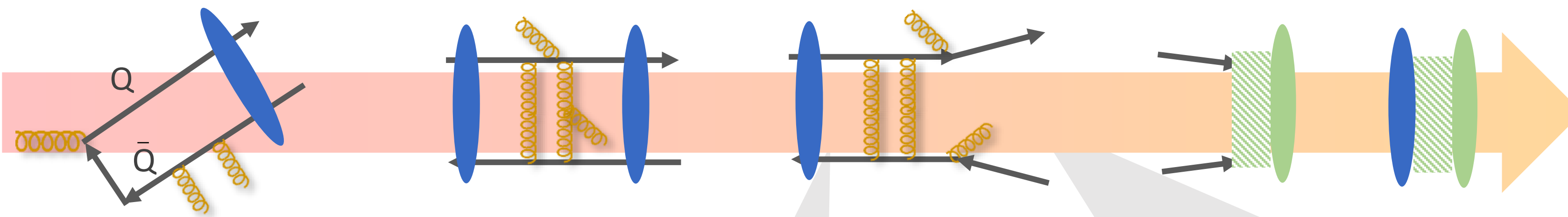

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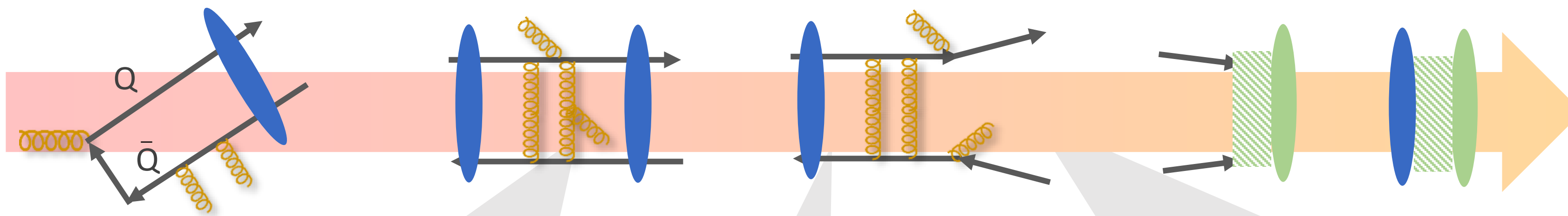

Real-time QQ evol. in local thermal equilibrium

Beyond Schrödinger:

Open-quantum-systems Lindblad equation

Connecting OQS to EFT language of potential

with Y.Akamatsu et.al. JHEP 1807 (2018) 029
with S.Kajimoto et.al. PRD97 (2018) 014003
with T. Miura et.al. (in progress)

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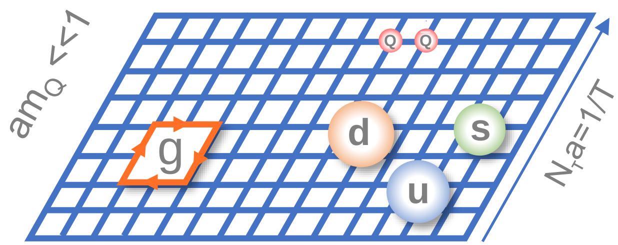
- Exploit $\frac{T}{m_Q} \ll 1, \ \frac{\Lambda_{QCD}}{m_Q} \ll 1$ to treat heavy quarks non-relativistically



Lattice QCD simulation with $Q\bar{Q}$
still too costly for bottom quarks

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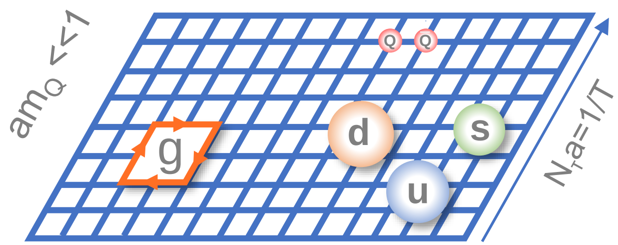

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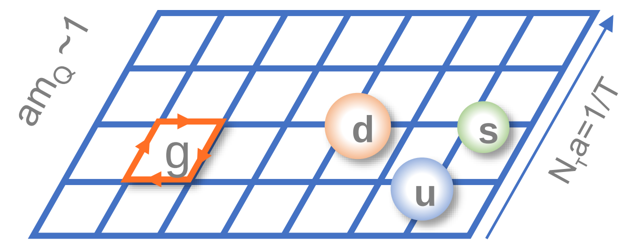

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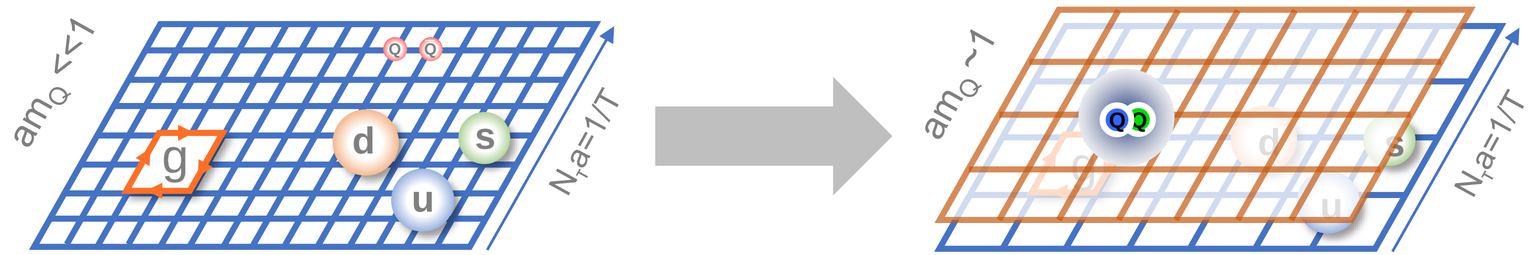
QQ̄ in NRQCD effective theory



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Lattice QCD simulation without QQ̄

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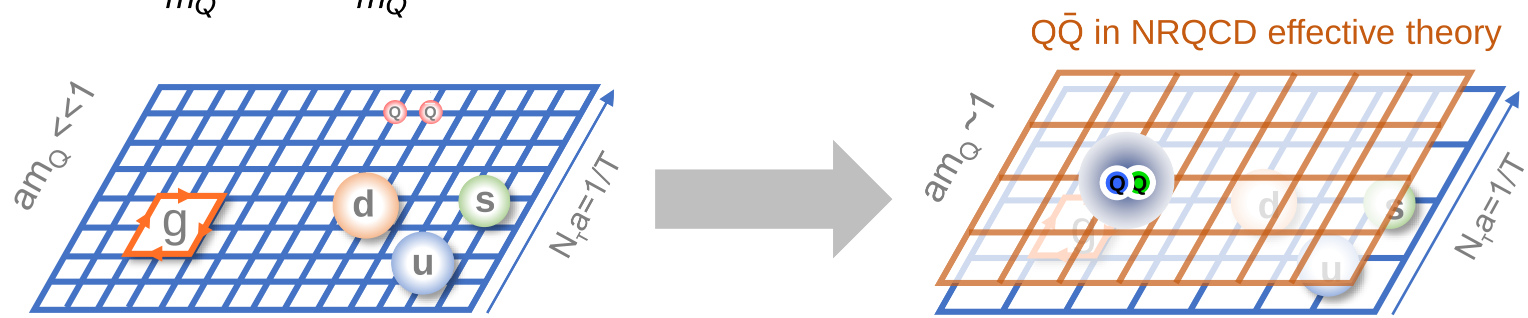

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  - our implementation uses $O(1/(m_Q a)^3)$ and leading order Wilson coefficients
- **Realistic** N_f=2+1 HISQ lattices for the QCD medium by HotQCD HotQCD PRD85 (2012) 054503, PRD90 (2014) 094503
  - **m_π=161MeV** T=0: N_τ=32-64 T>0: 48^3x12
    T= [**140 – 407**] MeV $m_b a$= [2.759 – 1.559]
    T= [140 – 251] MeV $m_c a$= [0.757 – 0.427]
    use adaptive step size to stabilize NRQCD expansion

The direct reconstruction challenge

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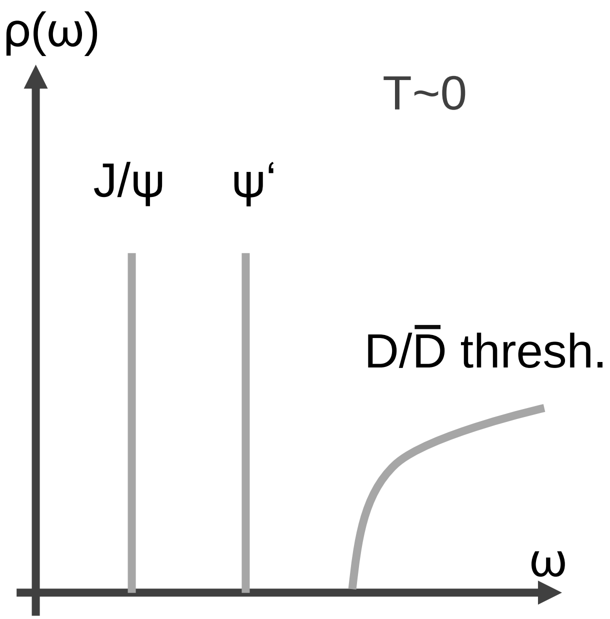
- **Lattice QCD** simulations are similar to a (very) **imperfect detector**



Quarkonium spectral function

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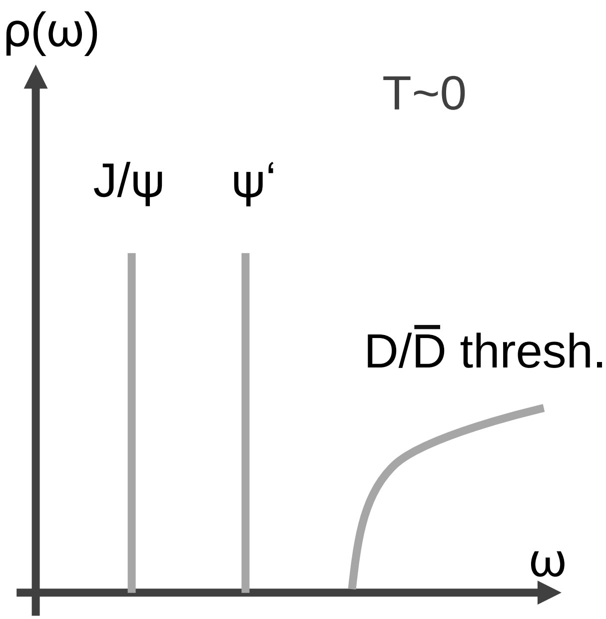

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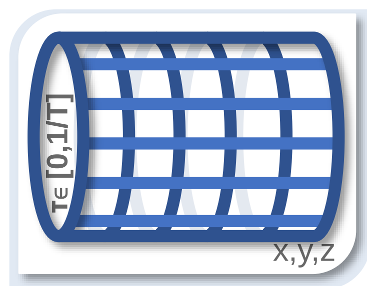
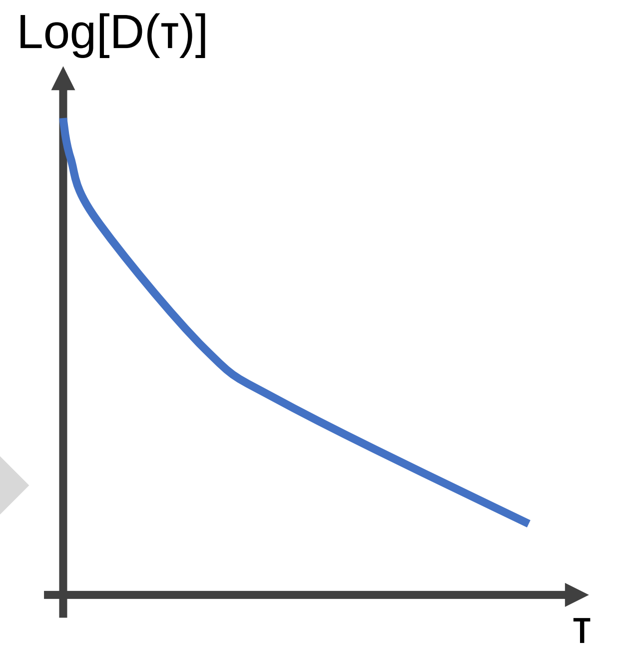


$$D(\tau) = \int_{-2M_Q}^{\infty} d\omega e^{-\tau\omega} \rho(\omega)$$



Euclidean time correlation function

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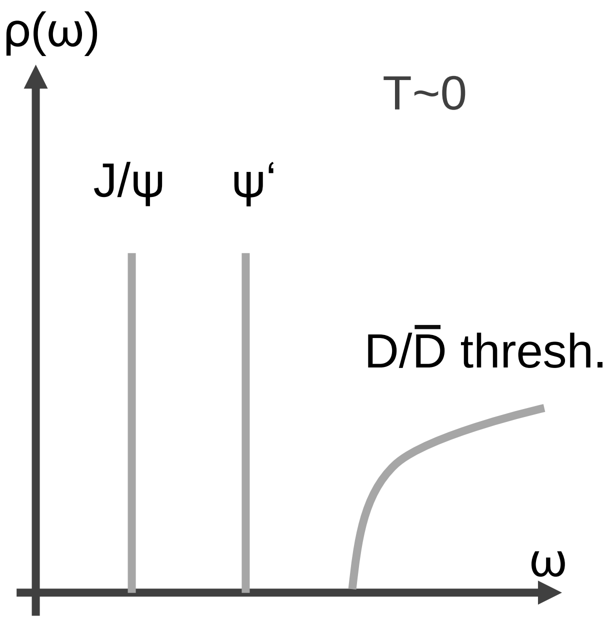

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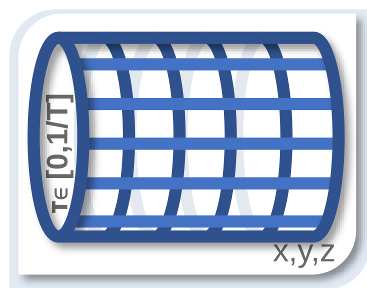

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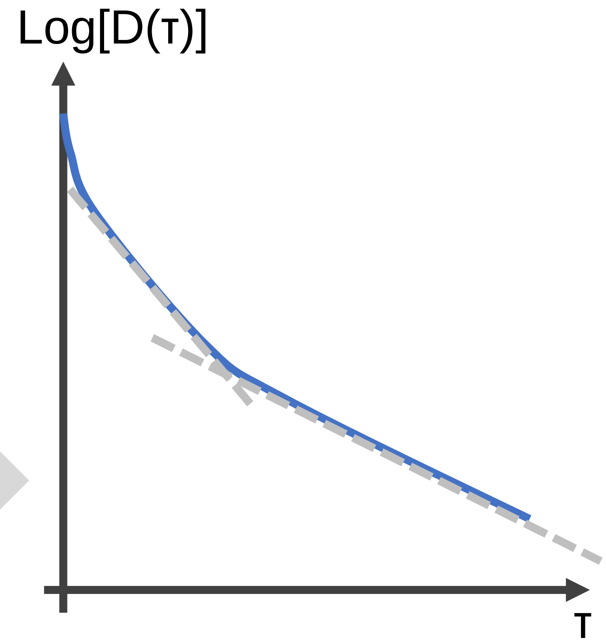

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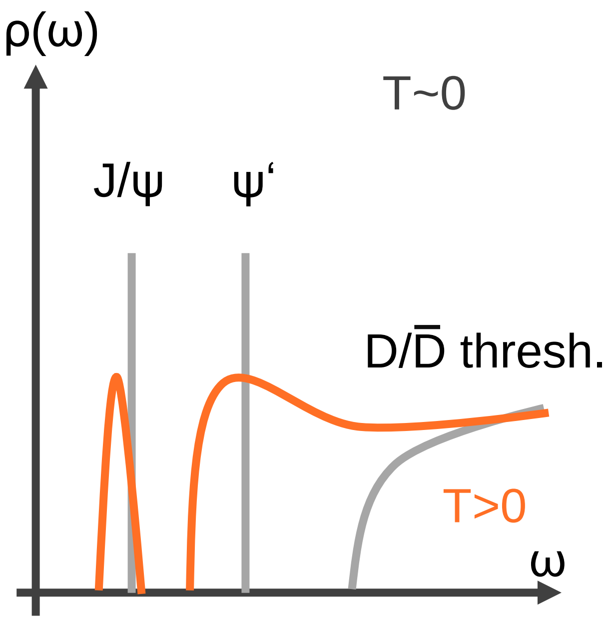

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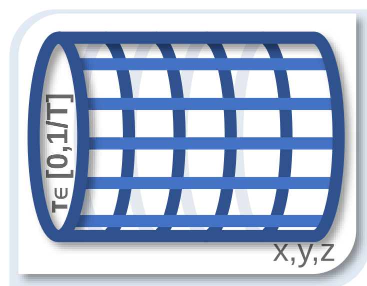

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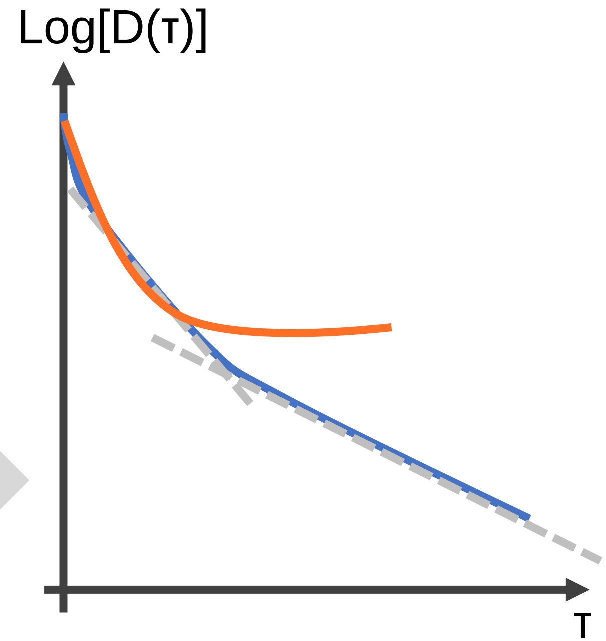

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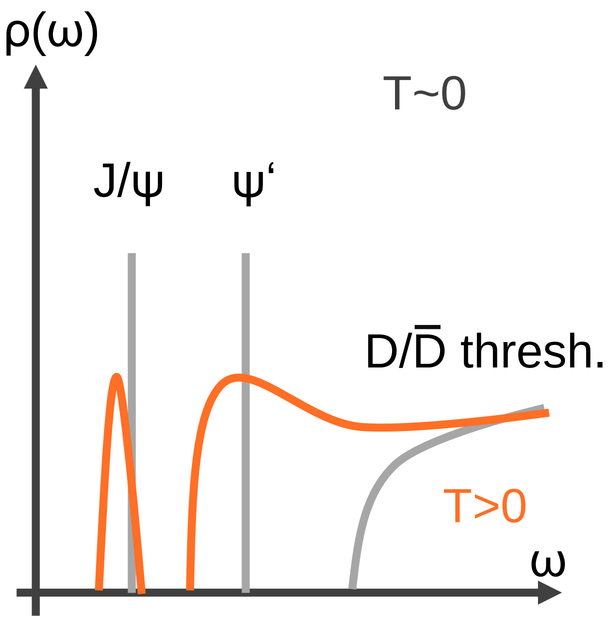

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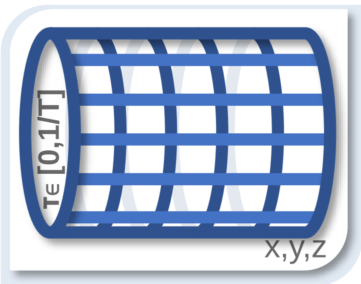

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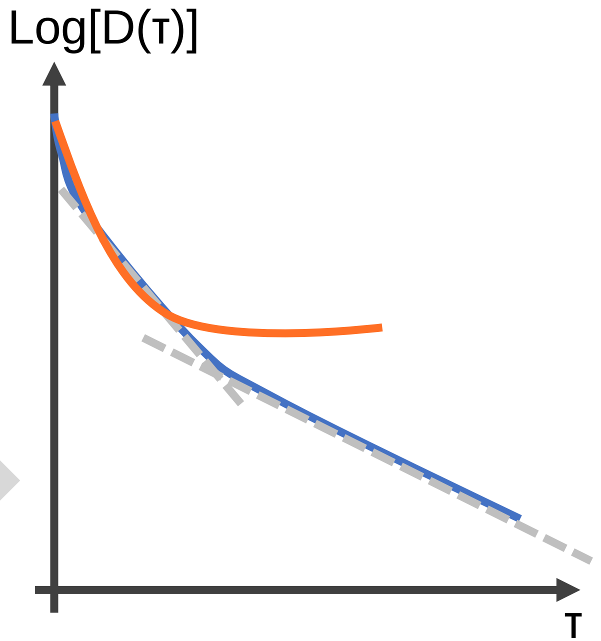

Euclidean time correlation function

- Extraction of spectra ill-posed **unfolding problem**: here via Bayesian inference

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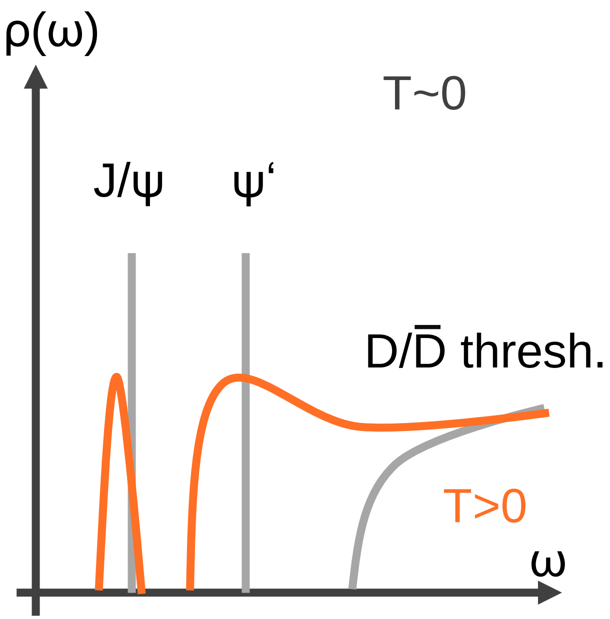

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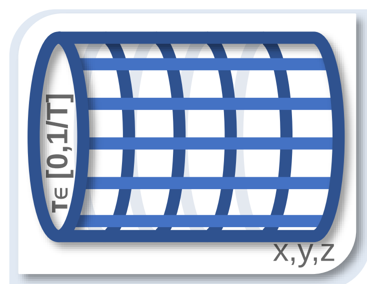

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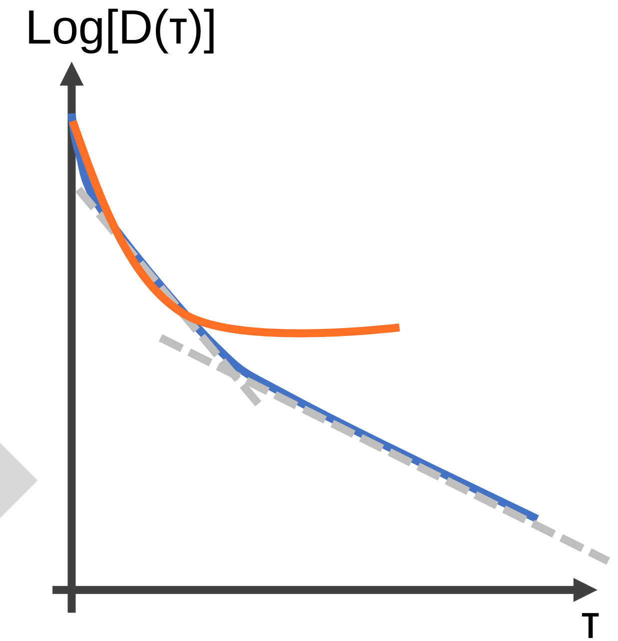

Euclidean time correlation function

- Extraction of spectra ill-posed **unfolding problem**: here via Bayesian inference
- Access to Euclidean time diminishes as T increases - different artifacts as @ T=0

Bayesian spectral reconstruction

- Inversion of Laplace transform required – highly ill-posed

$$D(\tau) = \int_{-2m_Q}^{\infty} d\omega e^{-\omega\tau} \rho(\omega)$$

Bayesian spectral reconstruction

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- Inversion of Laplace transform required – highly ill-posed

$$D_i = \sum_{l=1}^{N_\omega} \Delta\omega_l \, e^{-\omega_l \tau_i} \rho_l$$

1. N_ω parameters ρ_l >> N_τ datapoints
2. data D_i has finite precision

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- Regularize this task using prior information – Bayes introduces prior P[ρ|I]=exp[S]

M. Jarrell, J. Gubernatis, Physics Reports 269 (3) (1996)

$$P[\rho|D,I] \propto P[D|\rho,I]P[\rho|I] \qquad \left.\frac{\delta P[\rho|D,I]}{\delta\rho}\right|_{\rho=\rho^{\mathrm{BR}}} = 0$$

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Y.Burnier, A.R.
PRL 111 (2013) 18, 182003

$$S_{BR} = \alpha \int d\omega \left(1 - \frac{\rho}{m} + log\left[\frac{\rho}{m}\right]\right)$$

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C.Fischer, J. Pawlowski, A.R., C. Welzbacher PRD98 (2018) 014009

$$S_{BR}^{smooth} = \alpha \int d\omega \left(\kappa \left(\frac{\partial\rho}{\partial\omega}\right)^2 + 1 - \frac{\rho}{m} + log\left[\frac{\rho}{m}\right]\right)$$

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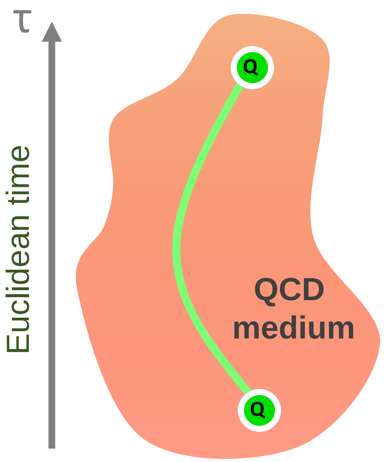
- Application of different approaches improves understanding of regularization artifacts

NRQCD Euclidean correlators



Non-rel. propagator of
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Davies, Thacker Phys.Rev. D45 (1992)

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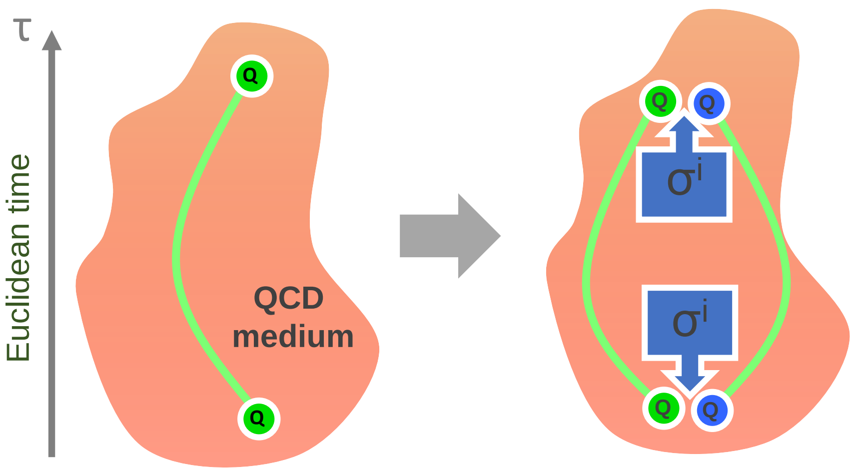

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QQ propagator
projected to a certain channel

„correlator of QQ wavefct.
$D_{J/\psi}(\tau) \triangleq \langle\psi_{J/\psi}(\tau)\psi^{\dagger}_{J/\psi}(0)\rangle$ "

Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

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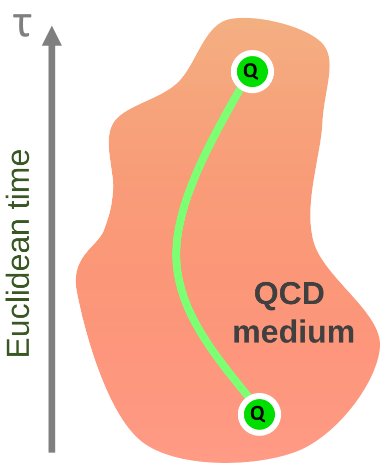

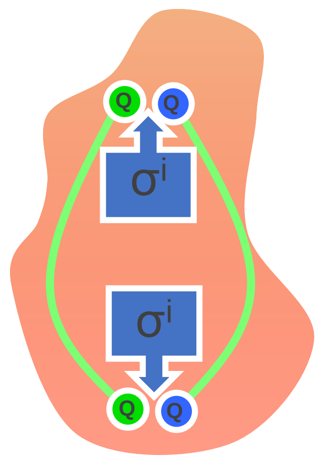

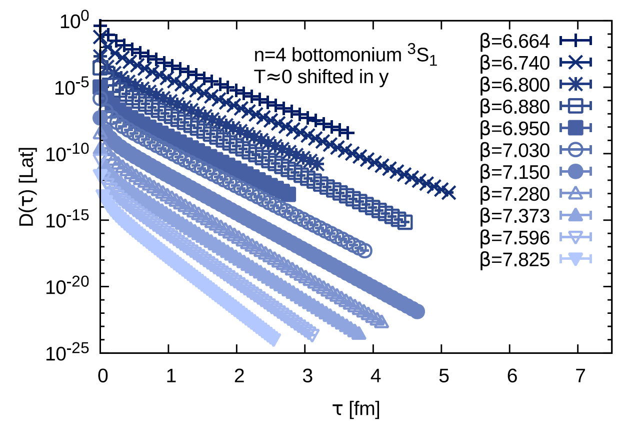

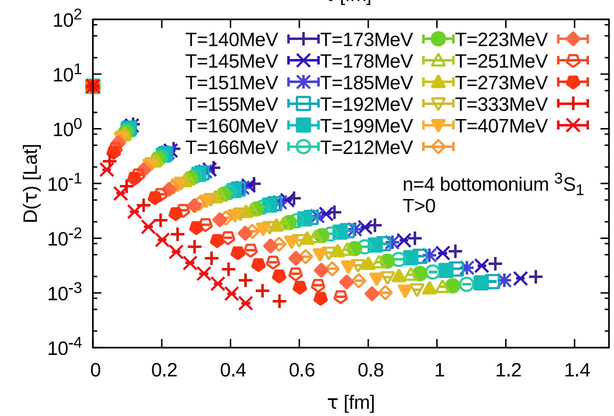

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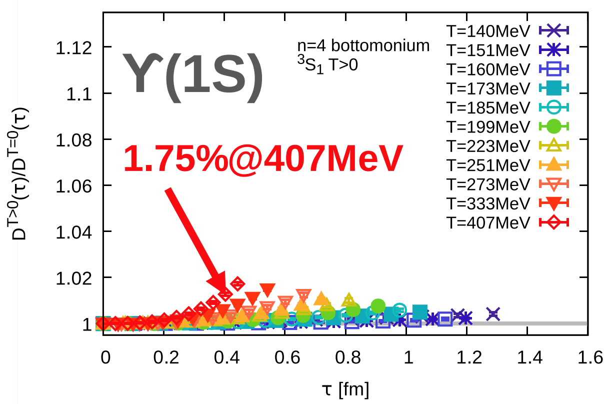
Euclidean correlation functions at T=0 and T>0

Correlator ratios (T>0 vs T=0)

raw lattice NRQCD results

E_{bind} (T=0)~1.1GeV



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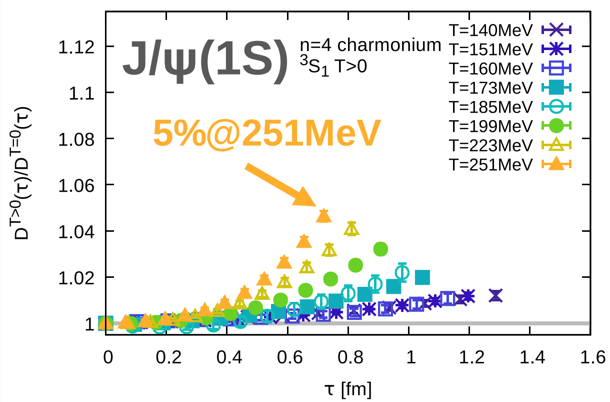

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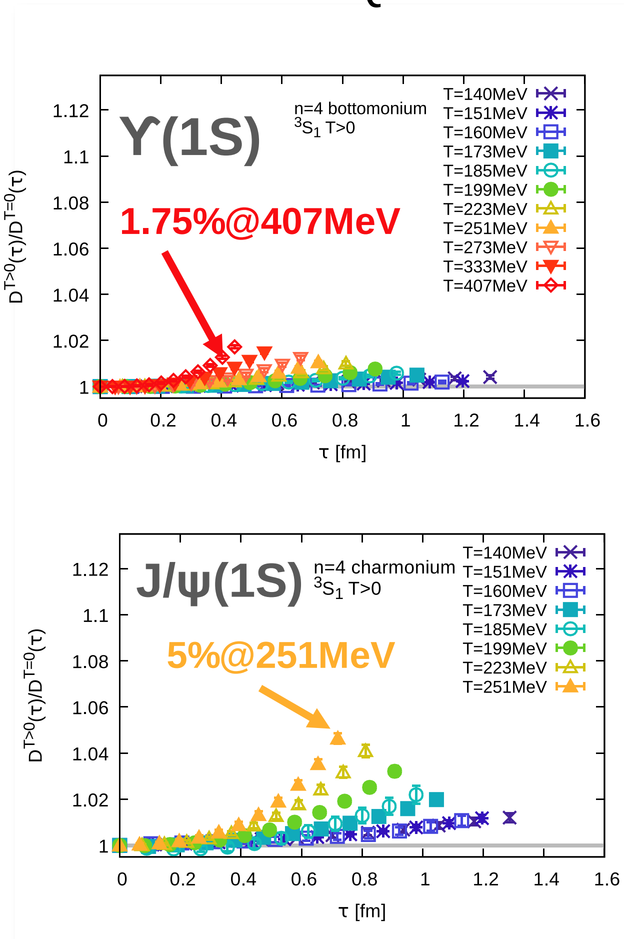

- In-medium modification hierarchically ordered with vacuum binding energy

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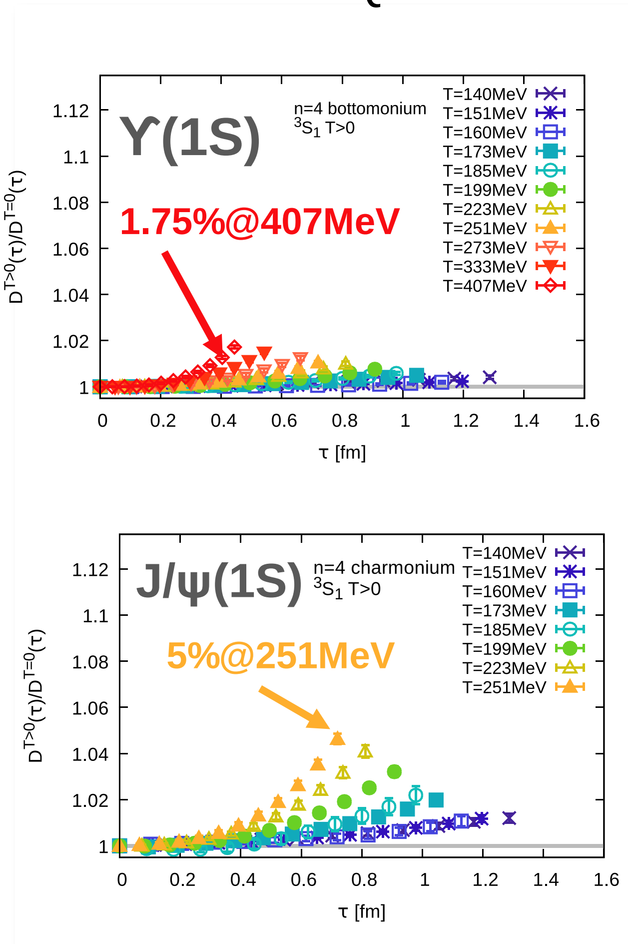
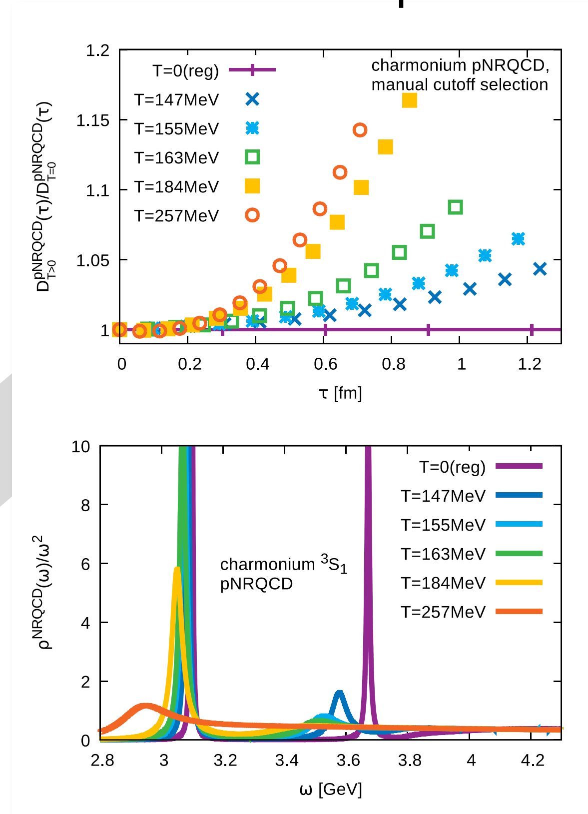
E_{bind} (T=0)~1.1GeV

E_{bind} (T=0)~0.6GeV



Correlator ratio approximated from the lattice potential

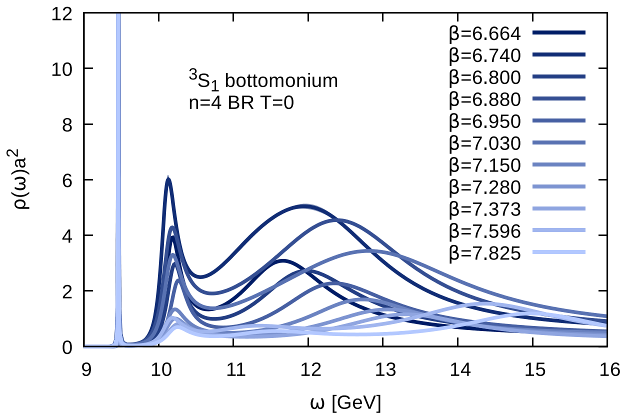


- In-medium modification hierarchically ordered with vacuum binding energy
- Upward bend compatible with lower in-medium mass (also seen in previous studies)

Lattice NRQCD spectral functions

University of Stavanger

$b\bar{b}$ S-wave T=0 spectra



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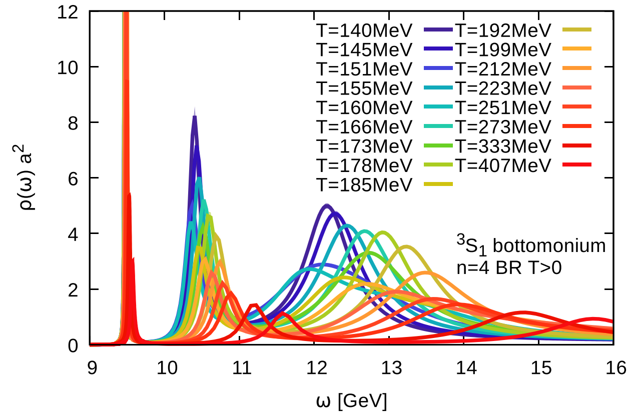

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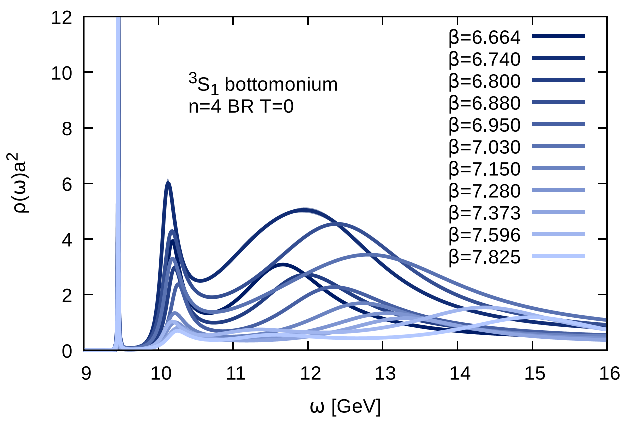

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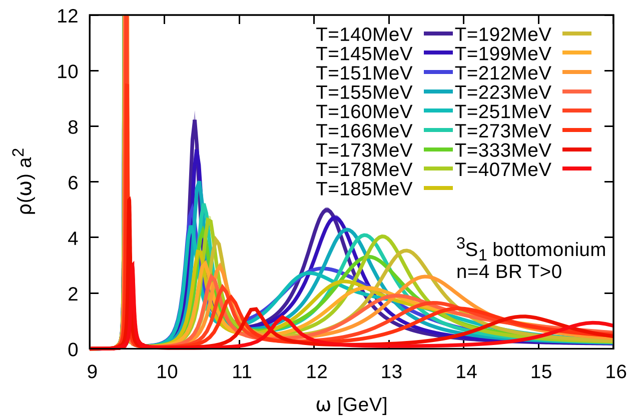

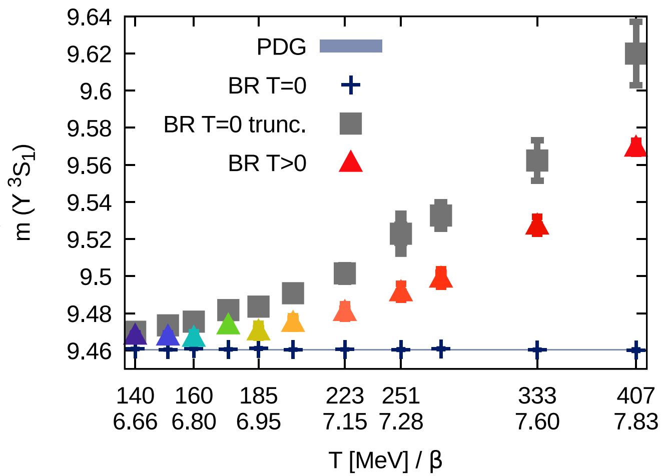

- Crucial step: **defining correct T=0 baseline** in presence of methods artifacts

Lattice NRQCD spectral functions

University of Stavanger

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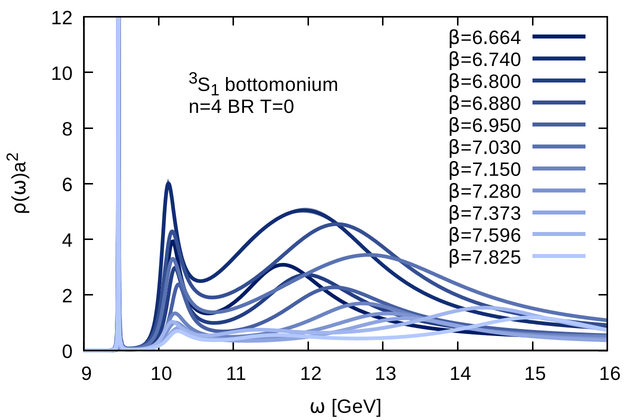

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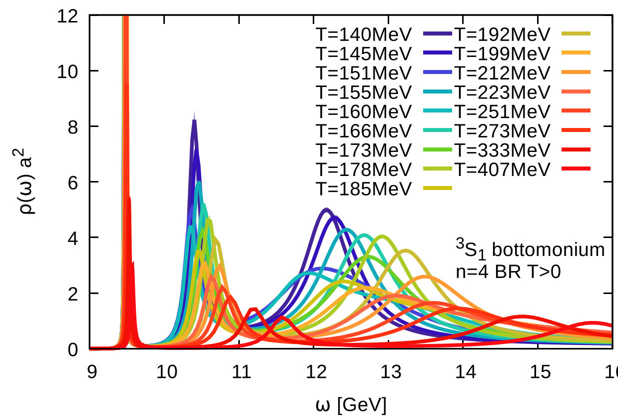
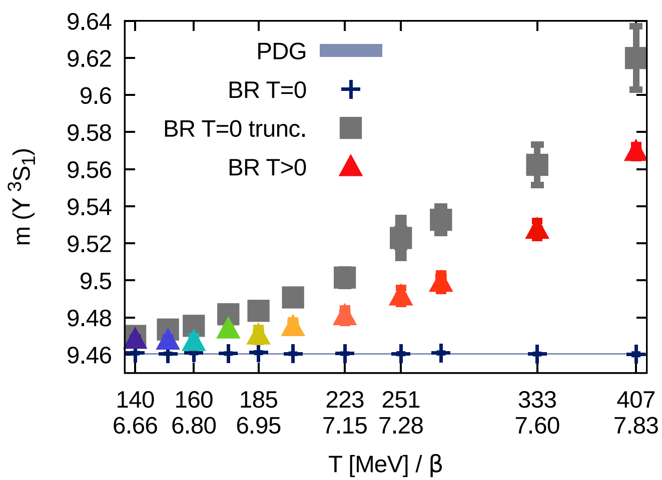
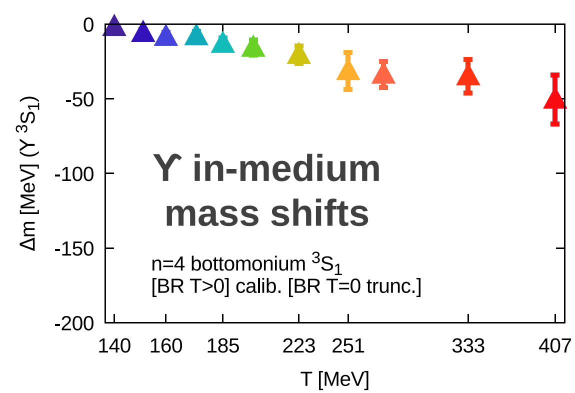




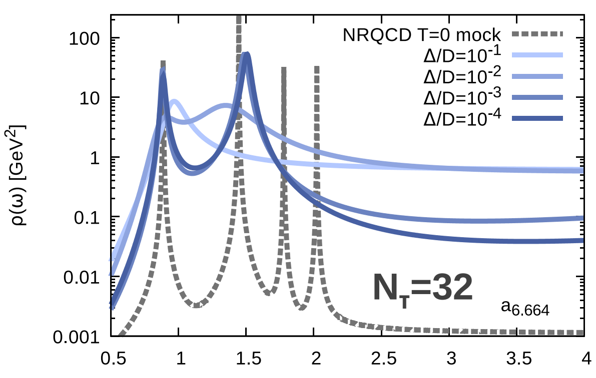


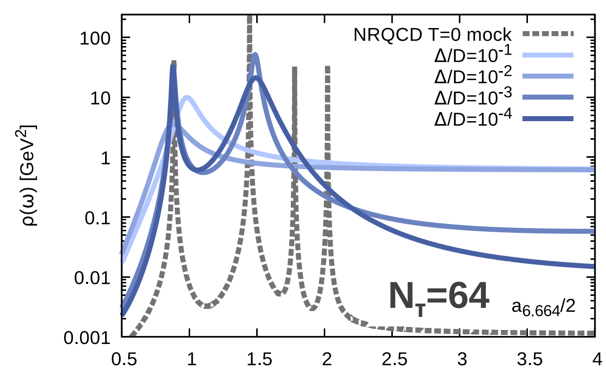
- Crucial step: **defining correct T=0 baseline** in presence of methods artifacts
- For the first time **consistent negative in medium mass shifts** – ordered by E_{bind}

How to improve in the future?

- Lattice community favorite strategy: more simulations @ smaller lattice spacing?

Mock Tests





- **No significant improvement** of bound state reconstruction on **finer lattices**

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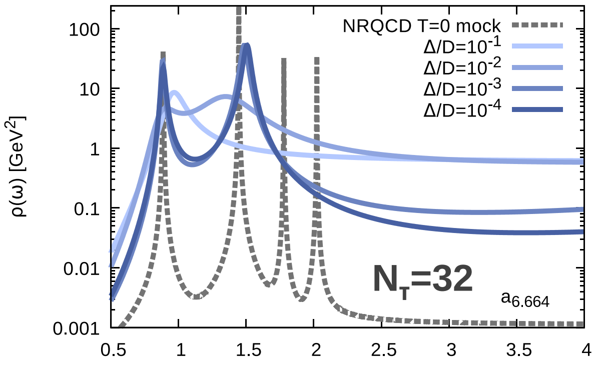
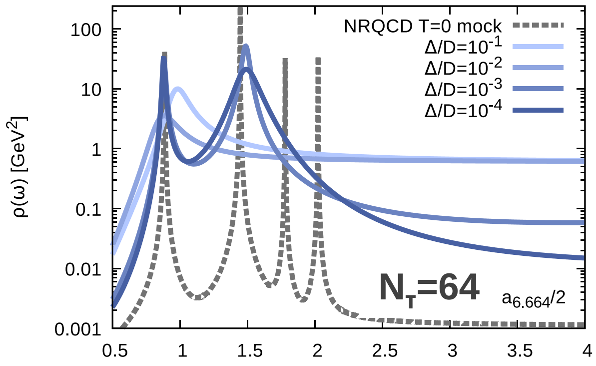




- **No significant improvement** of bound state reconstruction on **finer lattices**

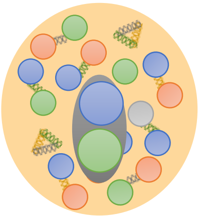
- Reached the onset of exponential difficulty: progress needs conceptually new ideas

The real-time interquark potential

University of Stavanger

- Exploit $\frac{T}{m_Q} \ll 1, \; \frac{\Lambda_{QCD}}{m_Q} \ll 1$ to treat heavy quarks non-relativistically

Relativistic T>0 field theory



Brambilla et.al. Rev.Mod.Phys. 77 (2005) 1423

QCD	NRQCD	pNRQCD
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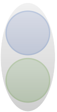

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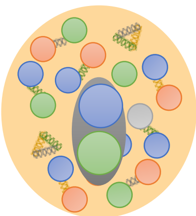

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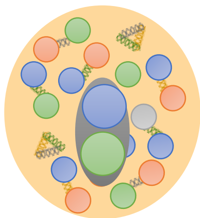

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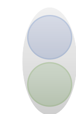

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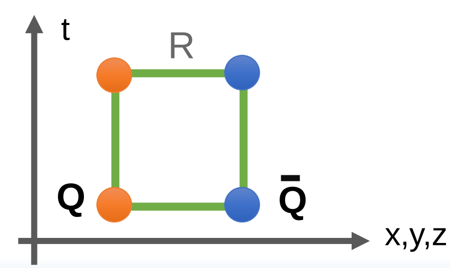

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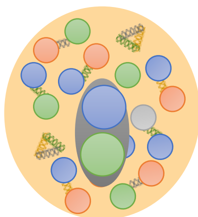

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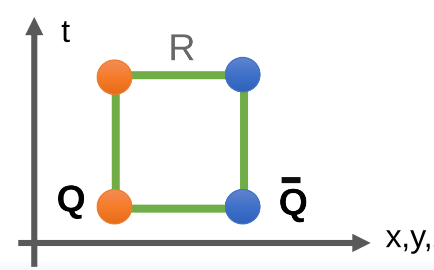

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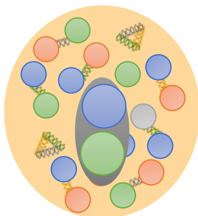

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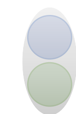

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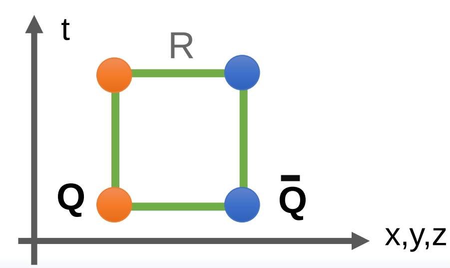
Im[V]: Laine et al. JHEP03 (2007) 054; Beraudo et. al. NPA 806:312,2008 Brambilla et.al. PRD 78 (2008) 014017



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University of Stavanger

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A.R., T.Hatsuda & S.Sasaki
PRL 108 (2012) 162001

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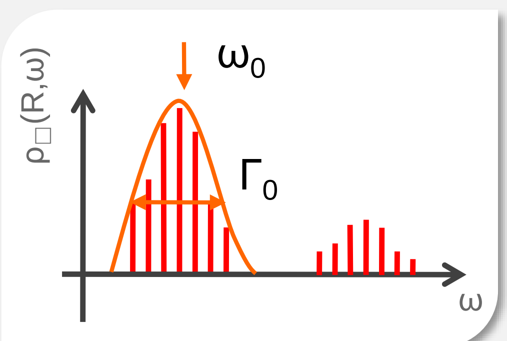

well defined V(R)
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For technical details see
Y.B., A.R. PRD86 (2012) 051503

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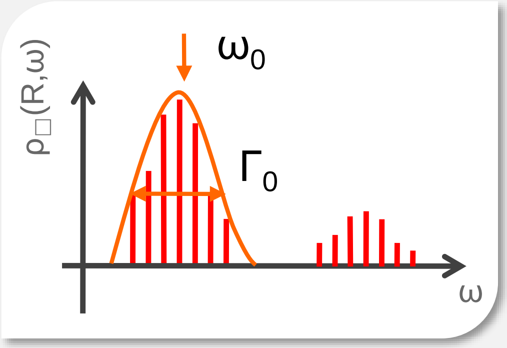

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$$V(R) = \omega_0(R) - i\Gamma_0(R)$$

Spectral Reconstruction

- In case of usual ΔW/W=10^{-2} statistical uncertainty in $W_\square$: **Bayesian inference**

incorporate prior information to regularize the inversion task (BR method)

- In case of **small ΔW/W<10^{-3}** statistical uncertainty in $W_\square$ also **Pade approximation**

exploit the analyticity of the Wilson correlator to extract spectra

Latest results on the lattice potential

University of Stavanger

- Lattices with dynamical u,d,s quarks (HISQ action, HotQCD & TUMQCD)

A. Bazavov et.al. PRD97 (2018) 014510, HotQCD PRD90 (2014) 094503

  - realistic m_π~161MeV (T=151-1451MeV)
  - fixed box (N_s=48 - **N_τ=12, N_τ=16**) & very **high statistics** 4000-9000 realizations
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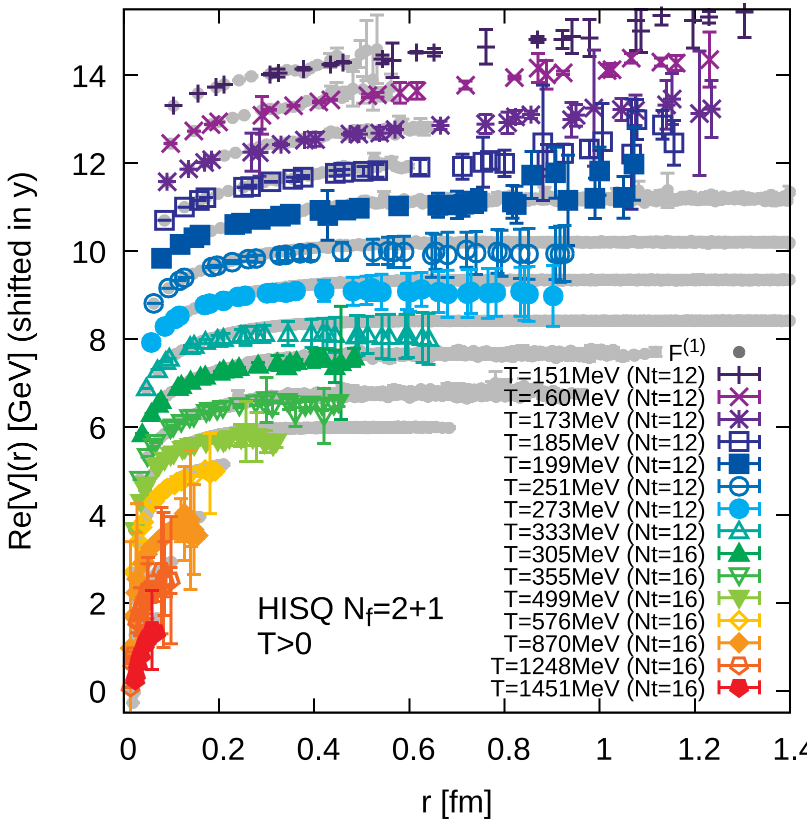

P. Petreczky, A.R., J. Weber, NPA982 (2019) 735

- Smooth transition from Cornell @ T=0 to Debye screened @ $T>T_C$

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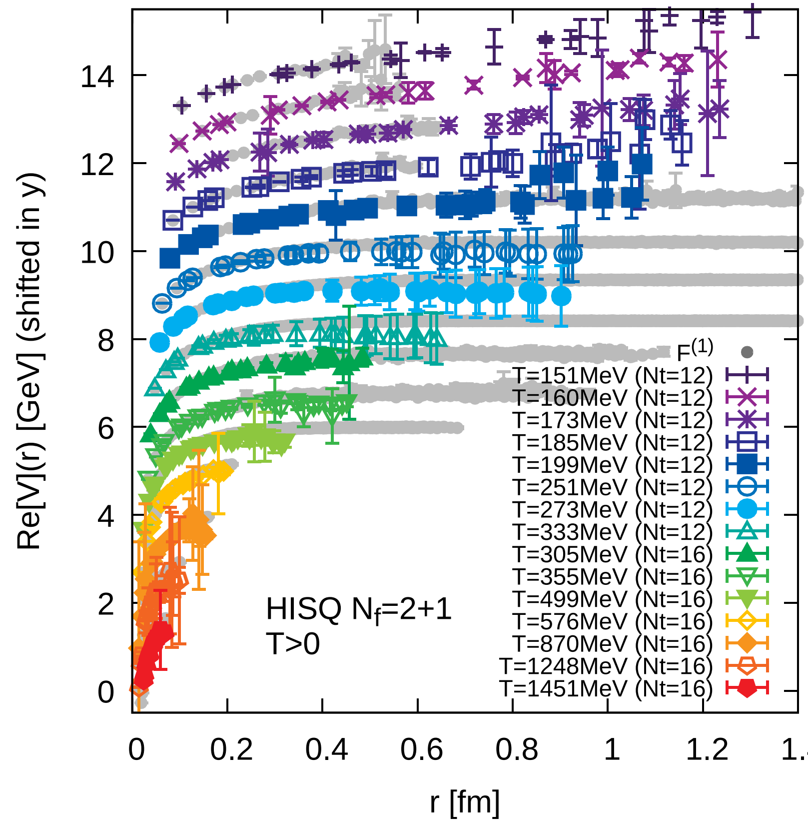

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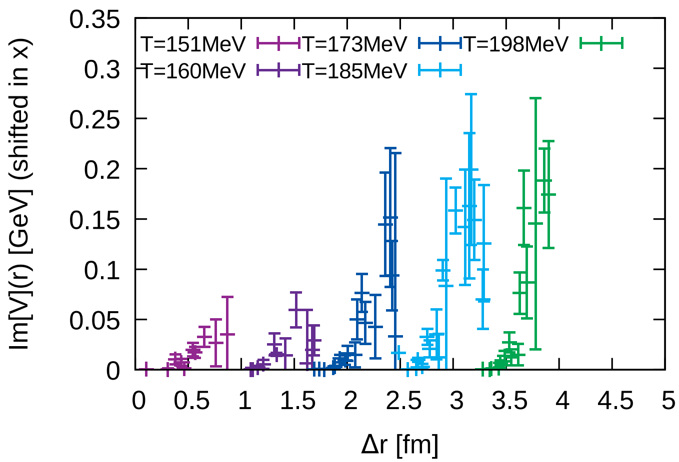
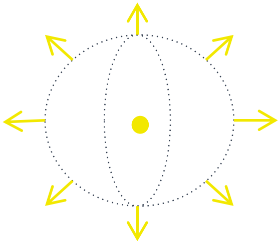


- Smooth transition from Cornell @ T=0 to Debye screened @ T>T_C
- Finite Im[V] above T_c present

An improved Gauss law approach

- For use in phenomenology applications: analytic expression for Re[V] and Im[V]



$$V_{Q\bar{Q}}^{T=0}(R) = V_C(R) + V_S(R) = -\frac{\alpha_S}{r} + \sigma r + c$$

Strategy:

α_s,σ and c are vacuum prop. and do not change with T

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University of Stavanger

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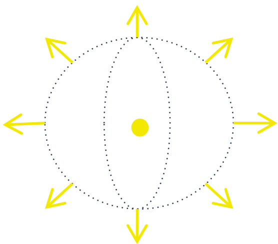

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Coulombic: a=-1   q=α_s

$$\vec{\nabla}\left(\vec{\nabla}V_C(R)\right) = -4\pi\alpha_S\delta(\vec{R})$$

String-like: a=+1 q=σ

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V. V. Dixit,
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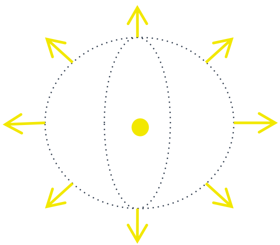

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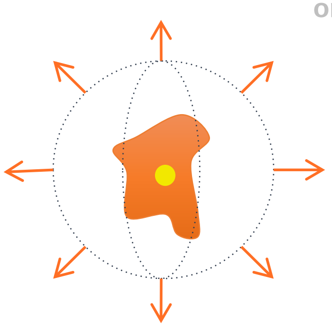
- Immerse non-perturbative charge in weak coupling HTL medium: permittivity ε

original idea: Y.Burnier, A.R. Phys.Lett. B753 (2016) 232 improved derivation D.Lafferty and A.R. arXiv:1906.00035

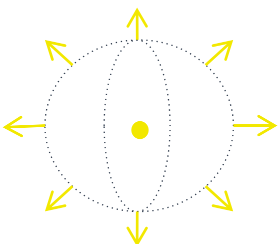


$$V^{med}(\mathbf{p}) = V^{vac}(\mathbf{p})/\epsilon(\mathbf{p}) \qquad \epsilon^{-1}(\vec{p}, m_D) = \frac{p^2}{p^2+m_D^2} - i\pi T \frac{p m_D^2}{(p^2+m_D^2)^2}$$

An improved Gauss law approach

University of Stavanger

- For use in phenomenology applications: analytic expression for Re[V] and Im[V]



$$V_{Q\bar{Q}}^{T=0}(R) = V_C(R) + V_S(R) = -\frac{\alpha_S}{r} + \sigma r + c$$

$$\mathcal{G}_a[V(R)] = \vec{\nabla}\left(\frac{\vec{\nabla}V(R)}{R^{a+1}}\right) = -4\pi q \delta^{(3)}(\vec{R})$$

Strategy:

α_s,σ and c are vacuum prop. and do not change with T

$$V(R) = aqR^a$$

Coulombic: a=-1 q=α_s

$$\vec{\nabla}\left(\vec{\nabla}V_C(R)\right) = -4\pi\alpha_S\delta(\vec{R})$$

String-like: a=+1 q=σ

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V. V. Dixit,
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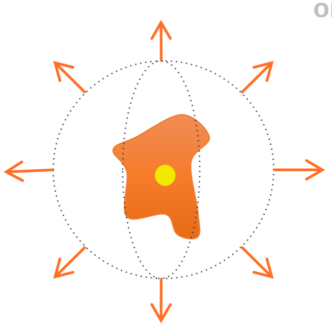

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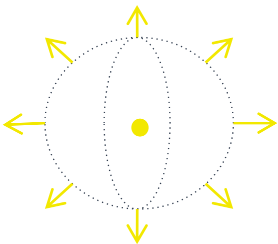

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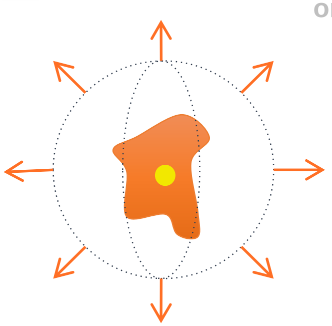

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- 3 vacuum parameters and 1 temperature dependent m_D fix both Re[V] and Im[V].

Gauss-law solution to Re[V] & Im[V]

- Gauss-Law result allows to fit Re[V] data even in the non-perturbative regime

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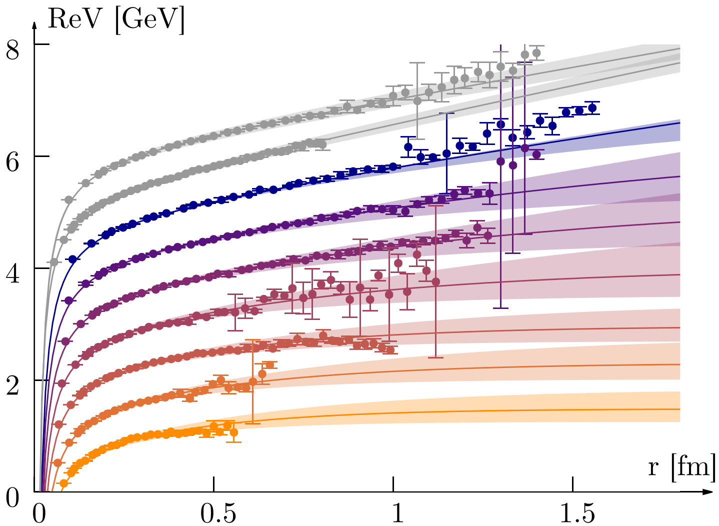

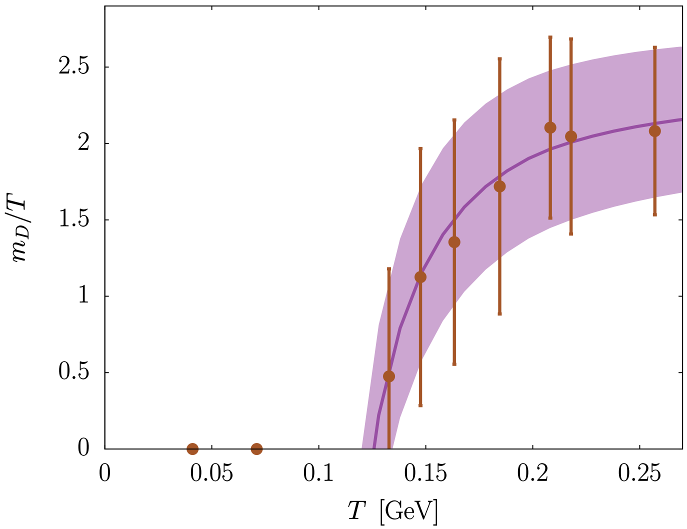

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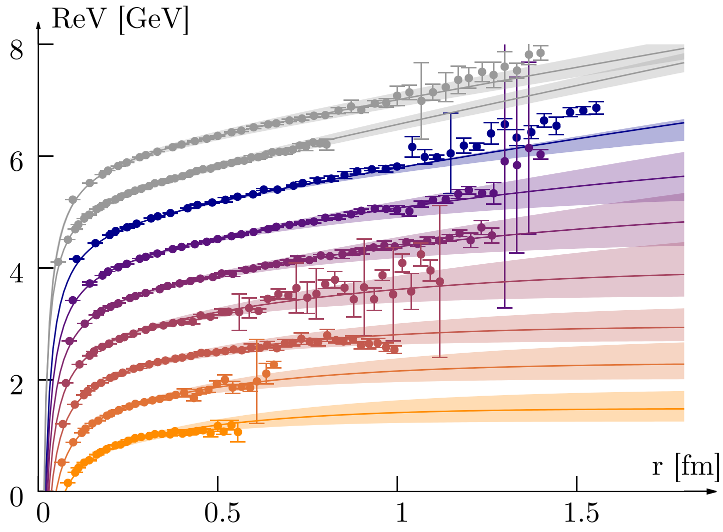

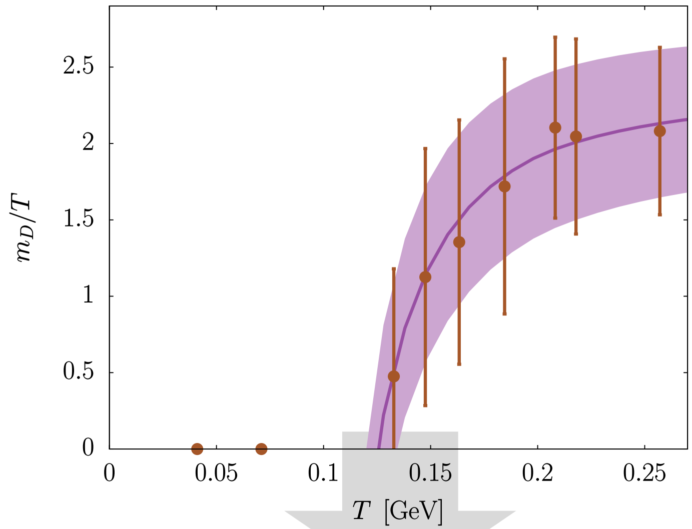

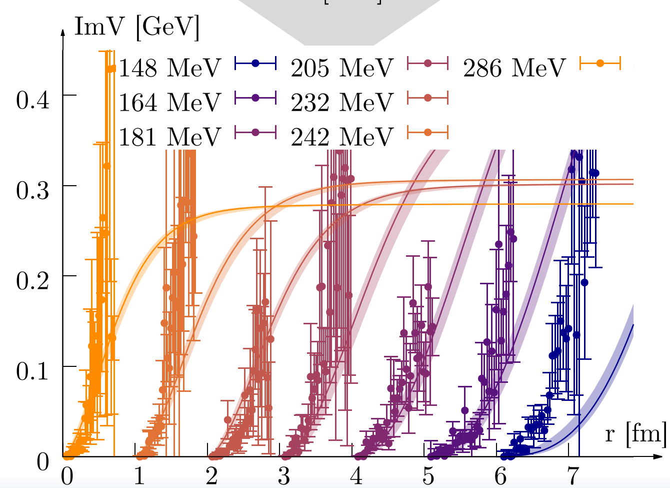

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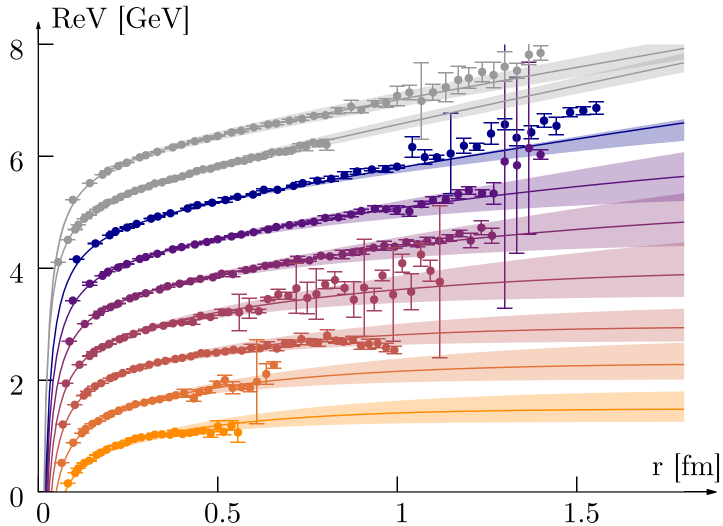

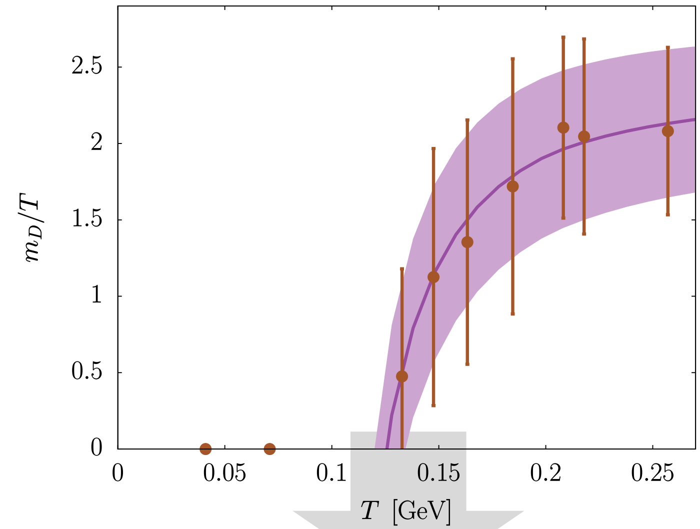

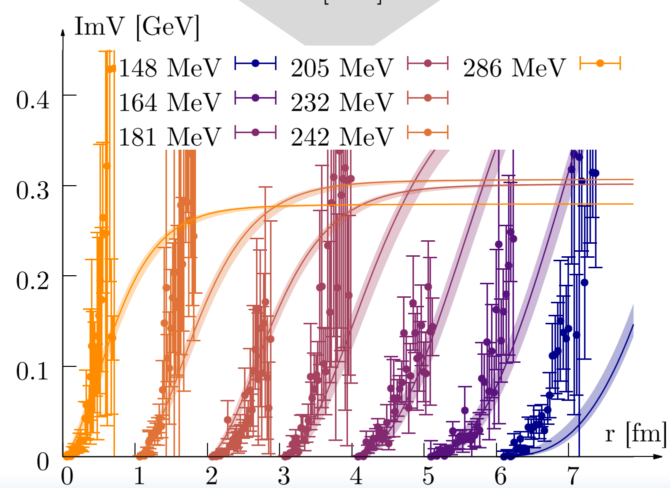

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- recently extend the Gauss law to model quarkonium at finite velocity & μ_B

The open quantum systems picture

- Need a general approach to describe quarkonium coupled to a thermal medium
  - Overall system is closed, hermitean Hamiltonian: von Neumann equation

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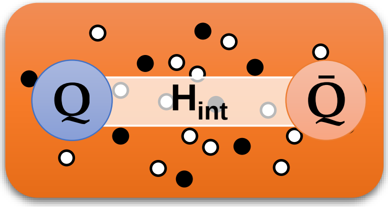

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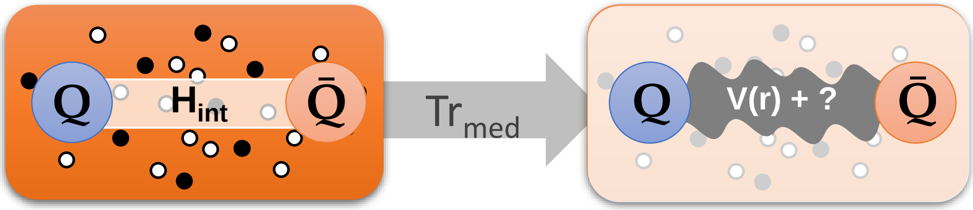

  - Dynamics of the reduced QQbar system: $\rho_{Q\bar{Q}} = \mathrm{Tr}_{med}\left[\rho\right] \qquad \frac{d\rho_{Q\bar{Q}}}{dt} = ?$

for an EFT result see Brambilla et.al. PRD96 (2017) , 034021

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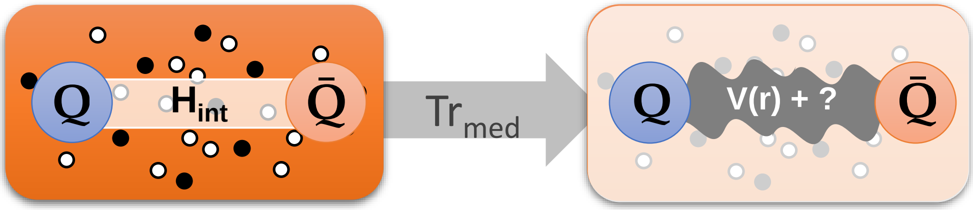

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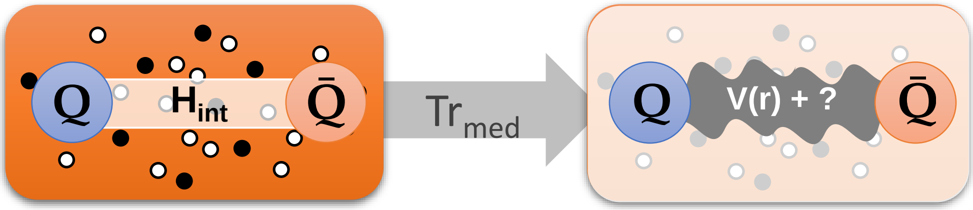

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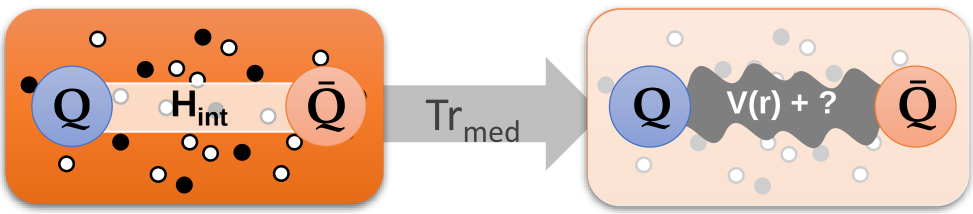

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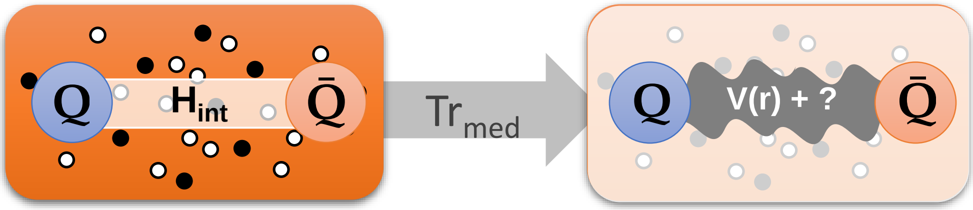

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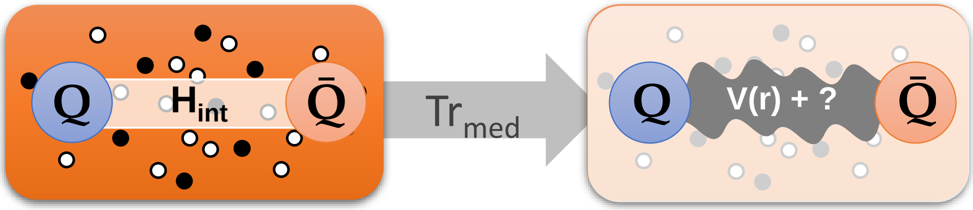

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medium - QQ interaction

The Lindblad equation

- Use scale separation: $m_Q > T$ heavy mass & weak coupling approximation

$$S_{FV} \approx S_{pot}\big[Re[V]\big] + S_{fluct}\big[Im[V]\big] + S_{diss}\big[Im[V]\big] + S_{LB}$$

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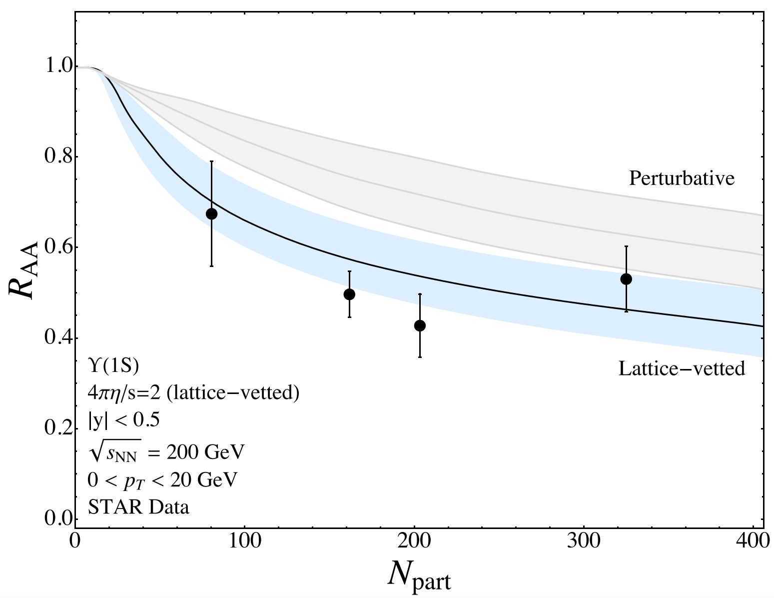
Krouppa et. al., PRDD97 (2018) 016017
[STAR Collaboration] Z. Ye
Nucl.Part.Phys.Proc. 289-290 (2017) 401



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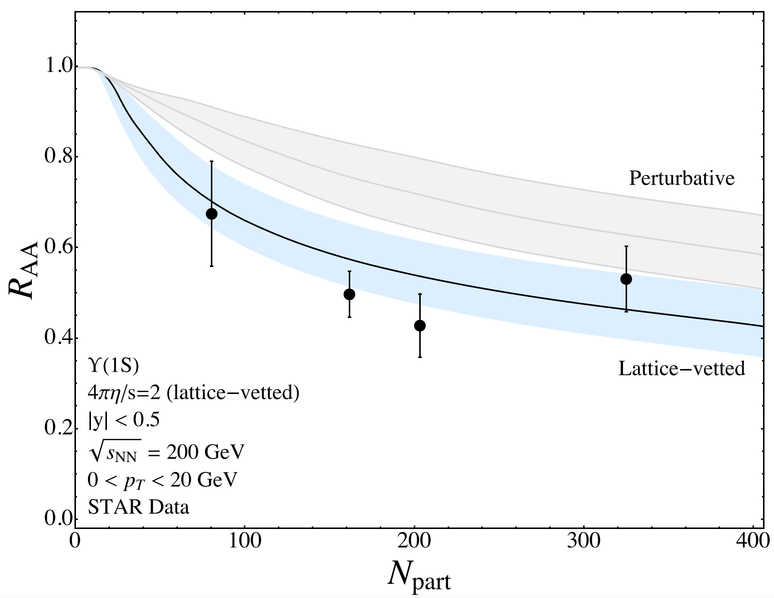
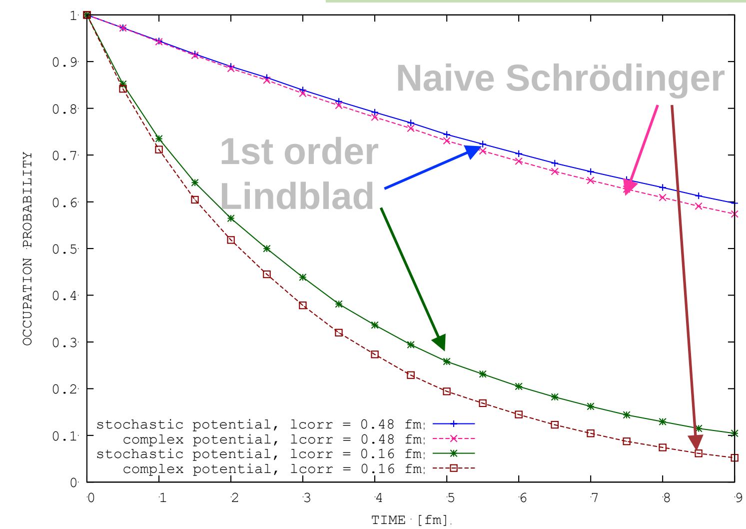


Comparison to 1st order approximation



S.Kajimoto, Y.Akamatsu, M. Asakawa,
A.R., PRD97 (2018) 014003

Quantum state diffusion

University of Stavanger

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- Actually: unravel into wavefunction stochastic dynamics: **Quantum State Diffusion**

T. Miura, Y.Akamatsu, M. Asakawa, S. Kajimoto, A.R., in preparation

$$\begin{aligned} |d\psi\rangle &= |\psi(t+dt)\rangle - |\psi(t)\rangle \\ &= -iH|\psi(t)\rangle dt + \sum_n \begin{pmatrix} 2\langle L_n^\dagger\rangle_\psi L_n - L_n^\dagger L_n \\ - \langle L_n^\dagger\rangle_\psi \langle L_n\rangle_\psi \end{pmatrix} |\psi(t)\rangle dt \\ &\quad + \sum_n \left(L_n - \langle L_n\rangle_\psi\right) |\psi(t)\rangle d\xi_n, \end{aligned}$$

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high temperature weak coupling parameters

Quantum state diffusion

University of Stavanger

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- First genuine Lindblad implementation: previous works could not maintain positivity of ρ

D. De Boni, JHEP 1708 (2017) 064

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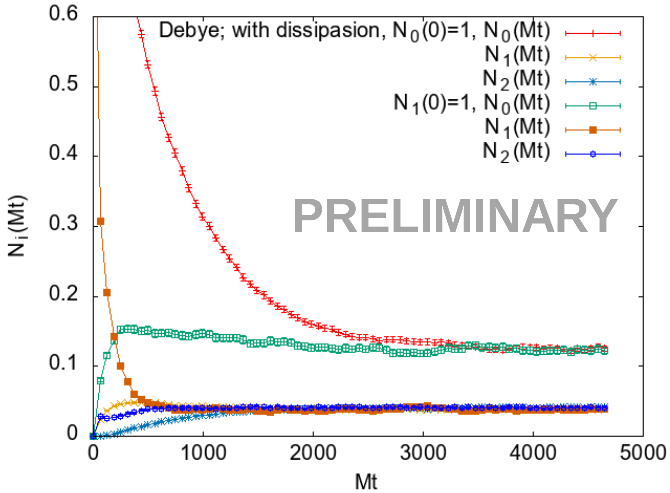
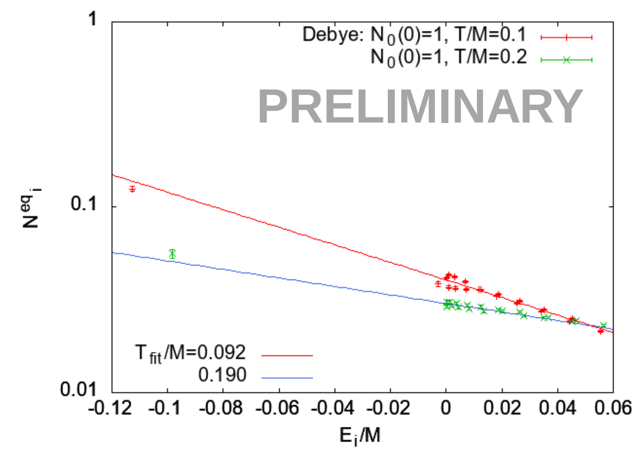
Proof of principle in 1d





- Encouraging preliminary results: admixtures become independent of initial conditions
- Distribution of states at late times agrees well with Boltzmann and yields consistent T

Conclusion

University of Stavanger

- **Conceptual** and **technical progress** in in-medium quarkonium theory
- Recent and ongoing studies on quarkonium dynamical properties
  - Control over systematics in direct **spectral reconstructions** in lattice NRQCD
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- A lot of work remains to be done:
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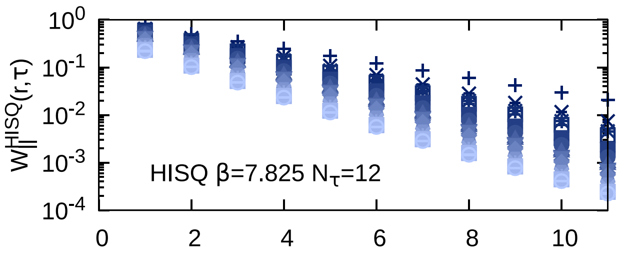
Thank you for your attention

Extracting the potential

- Example: Pade based reconstructions at β=7.825 T=407MeV N_τ=12

P. Petreczky, A.R. J. Weber in preparation



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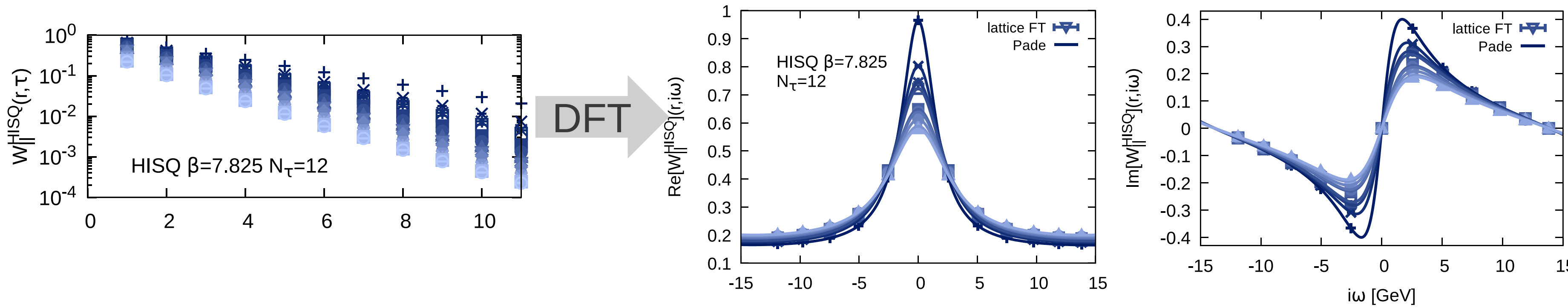

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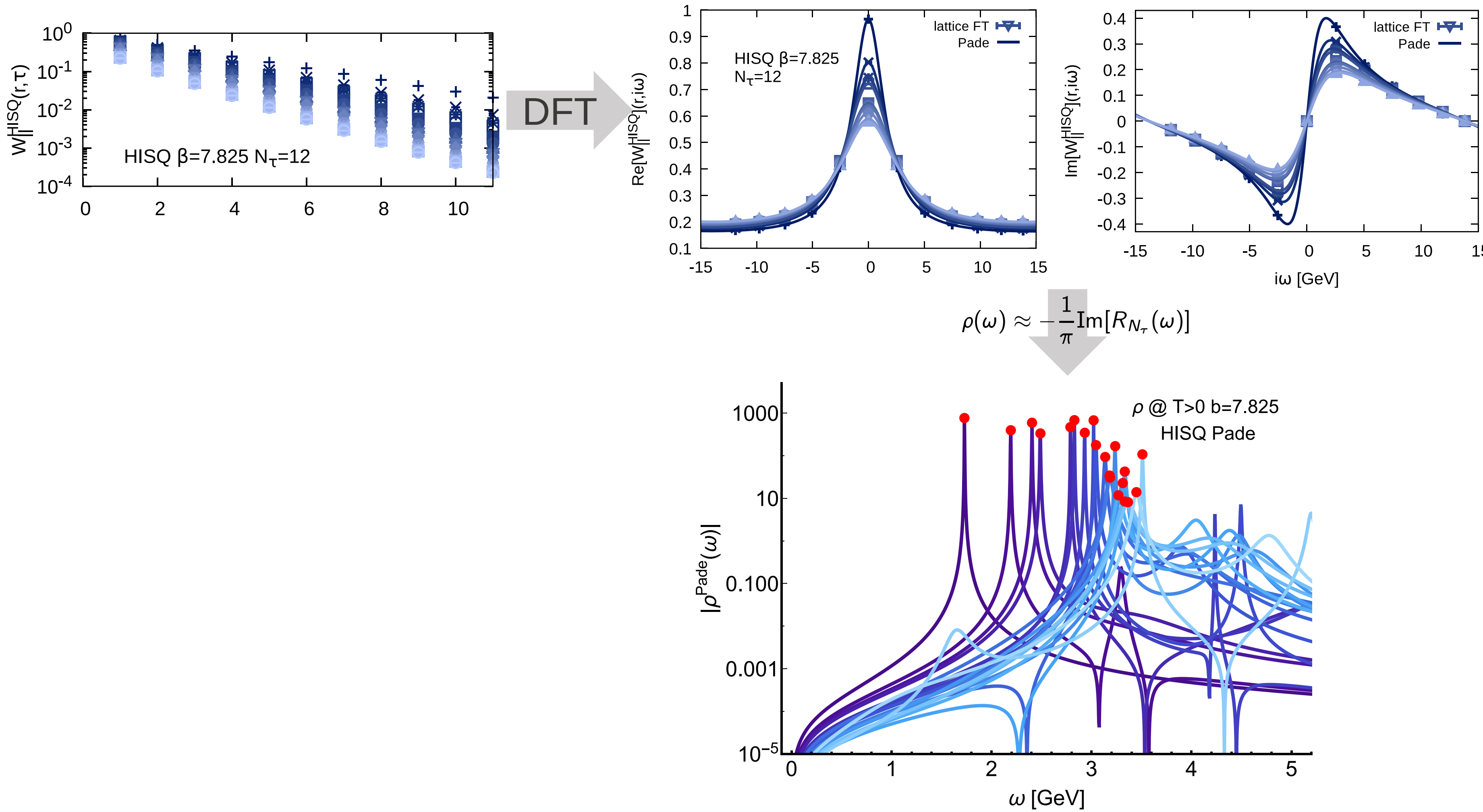

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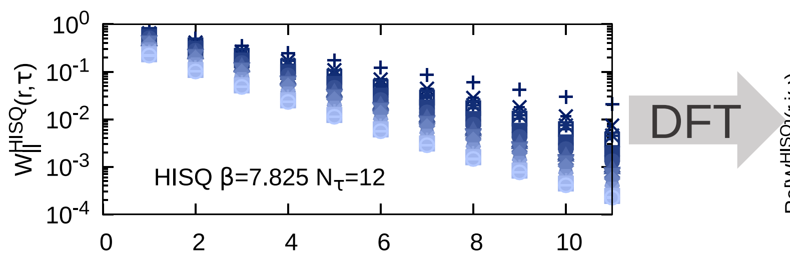
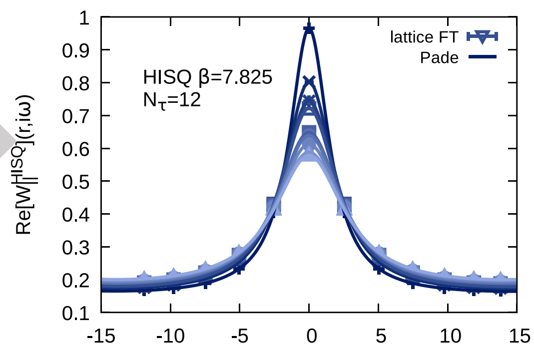
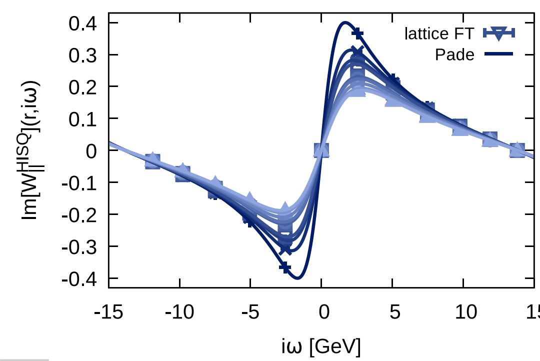
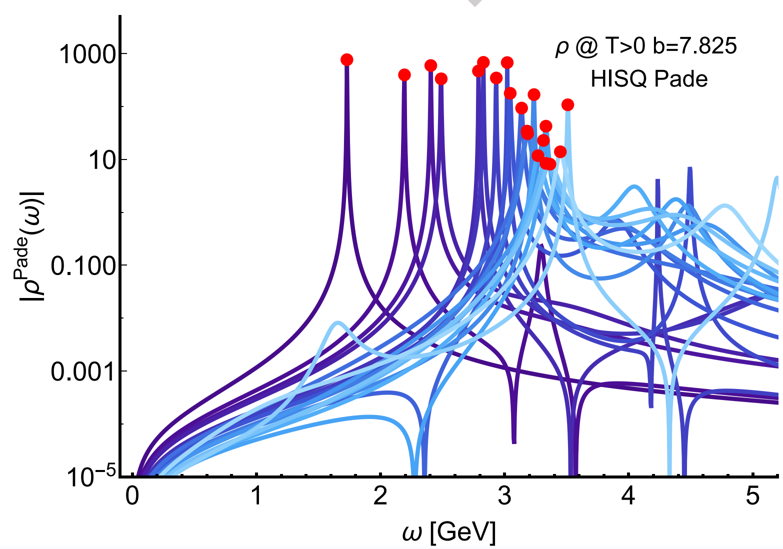






$$\rho(\omega) \approx -\frac{1}{\pi}\mathrm{Im}[R_{N_\tau}(\omega)]$$



- Always find well defined lowest peak: potential picture appears viable
- Beware of Pade artifacts besides peak: e.g. positivity violation, spikes

Stochastic potential

- Use scale separation: $m_Q > T$ heavy mass & weak coupling approximation

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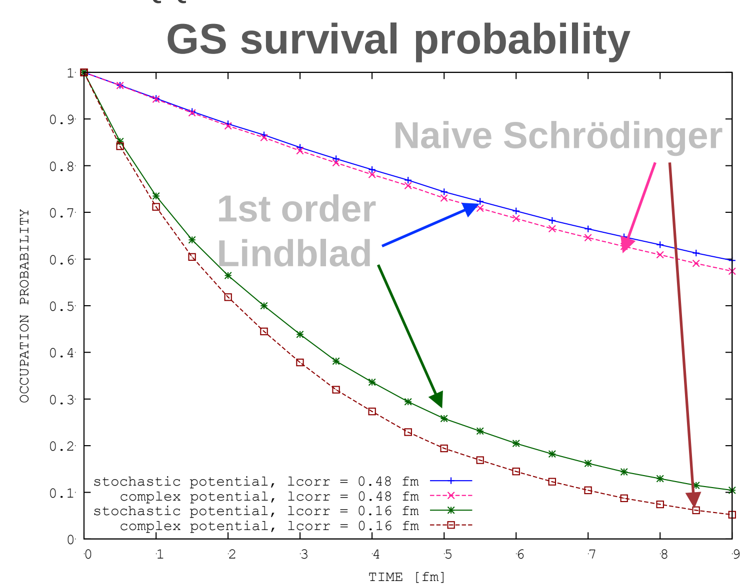

S.Kajimoto, Y.Akamatsu, M. Asakawa, A.R., PRD97 (2018) 014003

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S.Kajimoto, Y.Akamatsu, M. Asakawa, A.R., PRD97 (2018) 014003

GS survival probability